

1.

(1)

$$\begin{aligned}
 f(t) &= \frac{1}{\sqrt{2}} \cos(t) - \frac{d}{dt} \cos\left(t - \frac{\pi}{4}\right) \\
 &= \operatorname{Re} \left[\frac{1}{\sqrt{2}} e^{jt} - \frac{d}{dt} e^{j\left(t - \frac{\pi}{4}\right)} \right] = \operatorname{Re} \left[\frac{1}{\sqrt{2}} e^{jt} - j e^{j\left(t - \frac{\pi}{4}\right)} \right] \\
 &= \operatorname{Re} \left[e^{jt} \left(\frac{1}{\sqrt{2}} - j e^{-j\frac{\pi}{4}} \right) \right] = \operatorname{Re} \left[e^{jt} \times \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{2}} \right] \\
 &= \frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{2}\right) = \frac{1}{\sqrt{2}} \sin(t)
 \end{aligned}$$

(2)

$$\frac{1}{2} \delta(t)$$

2.

$$(1) F^{-1} \left[\frac{e^{-j\omega}}{2 + j3\omega} \right] = \frac{1}{3} F^{-1} \left[\frac{e^{-j\omega}}{2/3 + j\omega} \right] = \frac{1}{3} e^{-\frac{2}{3}(t-1)} u(t-1)$$

$$(2) F^{-1} \left[\frac{1}{3 + j(\omega - 2)} \right] = e^{-3t} e^{j2t} u(t)$$

$$(3) F^{-1} \left[\frac{1}{(1 + j\omega)(2 + j\omega)} \right] = F^{-1} \left[\frac{1}{1 + j\omega} - \frac{1}{2 + j\omega} \right] = (e^{-t} - e^{-2t}) u(t)$$

3.

$$(1) \frac{1}{s^2}$$

$$(2) \frac{1}{(s+1)^2}$$

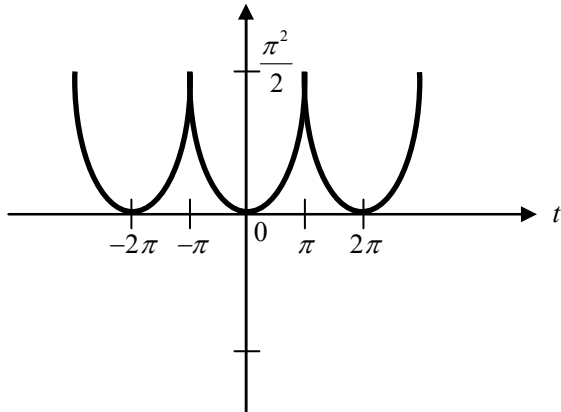
$$(3) \frac{s+1}{(s+1)^2 + 1}$$

4.

$$(1) (1 - e^{-2t}) \cdot u(t) \quad (2) F(s) = -\frac{2}{s+1} + \frac{3}{s+2} \quad f(t) = (-2e^{-t} + 3e^{-2t}) u(t)$$

$$(3) e^{-t} \cos 2t \cdot u(t)$$

5.
(1)



$$(2) a_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(t) dt = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{1}{2} t^2 dt = \frac{1}{\pi} \left[\frac{1}{6} t^3 \right]_{-\pi}^{\pi} = \frac{1}{3} \pi^2$$

(3) $\omega_0=1$ より

$$\begin{aligned} a_n &= \frac{2}{\pi} \int_0^{\pi} \frac{1}{2} t^2 \cos(nt) dt = \frac{2}{\pi} \left\{ \left[\frac{1}{2} t^2 \frac{\sin(nt)}{n} \right]_0^{\pi} - \int_0^{\pi} t \frac{\sin(nt)}{n} dt \right\} \\ &= \frac{2}{\pi} \left\{ \left[t \frac{\cos(nt)}{n^2} \right]_0^{\pi} - \int_0^{\pi} \frac{\cos(nt)}{n^2} dt \right\} \\ &= \frac{2}{\pi} \left\{ \frac{\pi}{n^2} (\cos n\pi) - \left(-\frac{1}{n} \left[\frac{1}{n} \sin(nt) \right]_0^{\pi} \right) \right\} \\ &= \frac{2}{n^2} (\cos n\pi) = \frac{2}{n^2} (-1)^n \end{aligned}$$

(4) 偶関数なので、 $b_n=0$

6.

ア. re^{-ar^2}

イ. $\frac{\pi}{a} e^{-z}$

ウ. $\frac{\pi}{a}$