電気数学Ⅱ　小テスト 12　解答例

2011.7.1　黒川

１．

(1) $v_i(t) = L\dfrac{di(t)}{dt} + Ri(t)$,　　　　$v_o(t) = Ri(t)$

(2) $V_i(\omega) = (j\omega L + R)I(\omega)$,　　　　$V_o(\omega) = RI(\omega)$

(3) $$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{R}{(j\omega L + R)} = \frac{1}{(1 + j\omega L/R)} = \frac{1}{\sqrt{1+(\omega L/R)^2}} e^{-j\tan^{-1}(\omega L/R)}$$

(4) $$\mathrm{h}(t) = F^{-1}\left[H(\omega)\right] = \left[\frac{R}{L} \cdot \frac{1}{(R/L + j\omega)}\right] = \frac{R}{L} e^{-\frac{R}{L}t} u(t)$$

(5) $v_i(t) = u(t)$ なので,

$$v_o(t) = h(t) * u(t) = \left[\int_0^t \frac{R}{L} e^{-\frac{R}{L}x} dx\right] u(t) = \left(1 - e^{-\frac{R}{L}t}\right) u(t)$$

２．

(1)

$$Y(\omega) = \frac{1}{20 + j\omega} = \frac{1}{\sqrt{400 + \omega^2}} \exp\left[-j\tan^{-1}\left(\frac{\omega}{20}\right)\right]$$

(2) $$e(t) = \sin 20t = \cos\left(20t - \frac{\pi}{2}\right) = \mathrm{Re}\left[\exp\left\{j\left(20t - \frac{\pi}{2}\right)\right\}\right]$$

$$i(t) = \mathrm{Re}\left[Y(20)\exp\left\{j\left(20t - \frac{\pi}{2}\right)\right\}\right] = \mathrm{Re}\left[\frac{1}{20\sqrt{2}} e^{-j\frac{\pi}{4}} \mathrm{e}^{j\left(20t - \frac{\pi}{2}\right)}\right] = \frac{1}{20\sqrt{2}} \cos\left(20t - \frac{3\pi}{4}\right)$$

(3) $e(t) = 1 + \cos 20t + \cos 60t$

$$\begin{aligned} i(t) &= \mathrm{Re}\left[Y(0)\exp(j0) + Y(20)\exp(j20t) + Y(60)\exp(j60t)\right] \\ &= \frac{1}{20} + \mathrm{Re}\left[\frac{1}{20\sqrt{2}} e^{-j\frac{\pi}{4}} \exp(j20t)\right] + \mathrm{Re}\left[\frac{1}{20\sqrt{10}} \exp\left\{-j\tan^{-1}(3)\right\} \exp(j60t)\right] \\ &= \frac{1}{20} + \frac{1}{20\sqrt{2}} \cos\left(20t - \frac{\pi}{4}\right) + \frac{1}{20\sqrt{10}} \cos\left(60t - \tan^{-1}(3)\right) \end{aligned}$$

(4) $e(t) = u(t)$　　　　この回路のインパルス応答 $y(t)$ は

$$y(t) = F^{-1}\left[Y(\omega)\right] = F^{-1}\left[\frac{1}{20 + j\omega}\right] = e^{-20t} u(t)$$

$$\begin{aligned} i(t) &= y(t) * u(t) \\ &= \left[\int_0^t e^{-20x} dx\right] \cdot u(t) = \left[-\frac{1}{20} e^{-20x}\right]_0^t \cdot u(t) = \frac{1}{20}\left(1 - e^{-20t}\right) \cdot u(t) \end{aligned}$$