

1. (各2点×7=14点)

$$(1) \int_{-\infty}^{\infty} (2t+1)^2 \delta(3t) dt = \frac{1}{3} (2t+1)^2 \Big|_{t=0} = \frac{1}{3}$$

$$(2) \int_{-\infty}^{\infty} \frac{\delta(t+2)}{t} dt = \frac{1}{t} \Big|_{t=-2} = -\frac{1}{2}$$

$$(3) (t^2+1)\delta(t-1) = (t^2+1) \Big|_{t=1} \delta(t-1) = 2\delta(t-1)$$

$$(4) (t-1)\delta(-3t) = (t-1) \Big|_{t=0} \frac{1}{3} \delta(t) = -\frac{1}{3} \delta(t)$$

$$(5) f(t) = \frac{1}{t+1} = (t+1)^{-1} \text{ とおくと、 } f'(t) = -\frac{1}{(t+1)^2} \text{ より}$$

$$\int_{-\infty}^{\infty} \frac{\delta'(t)}{(t+1)} dt = -f'(0) = -\left\{ -\frac{1}{(0+1)^2} \right\} = 1$$

$$(6) \int_{-\infty}^{\infty} \delta(t-7) dt = 1$$

$$(7) \int_{-\infty}^{\infty} e^{-t} \delta(t+2) dt = e^{-t} \Big|_{t=-2} = e^2$$

2. (各3点×2=6点)

$$(1) \int_{-\infty}^{\infty} (t+1)^2 \delta(t) dt = (t+1)^2 \Big|_{t=0} = 1, \quad (t-1)\delta(t-1) = (1-1)\delta(t-1) = 0$$

であるから、方程式は

$$\begin{aligned} t+1 &= 0 \\ \therefore t &= -1 \end{aligned}$$

$$(2) \int_{-\infty}^{\infty} \delta(t+2) dt = 1, \quad (t+1)\delta(t+1) = (-1+1)\delta(t+1) = 0$$

であるから、方程式は

$$\begin{aligned} 3t+1 &= 0 \\ \therefore t &= -\frac{1}{3} \end{aligned}$$