2011 電気数学Ⅱ　小テスト５　解答例

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図は軸ラベル、原点、交点の座標等を正しく記入すること．

(1)

$\sin 3t\cos 2t=\dfrac{1}{2}\left(\sin 5t+\sin t\right)$　sin5t の周期は $2\pi/5$、sin$t$ の周期は 2π、よって基本周期は 2π

(2)

$\exp\left(j\dfrac{2}{3}\pi\right)=\cos\dfrac{2}{3}\pi+j\sin\dfrac{2}{3}\pi=-\dfrac{1}{2}+j\dfrac{\sqrt{3}}{2}$　　(3) $1+j\sqrt{3}=2\left(\dfrac{1}{2}+j\dfrac{\sqrt{3}}{2}\right)=2\left(\cos\dfrac{\pi}{3}+j\sin\dfrac{\pi}{3}\right)=2e^{j\frac{\pi}{3}}$

(4)

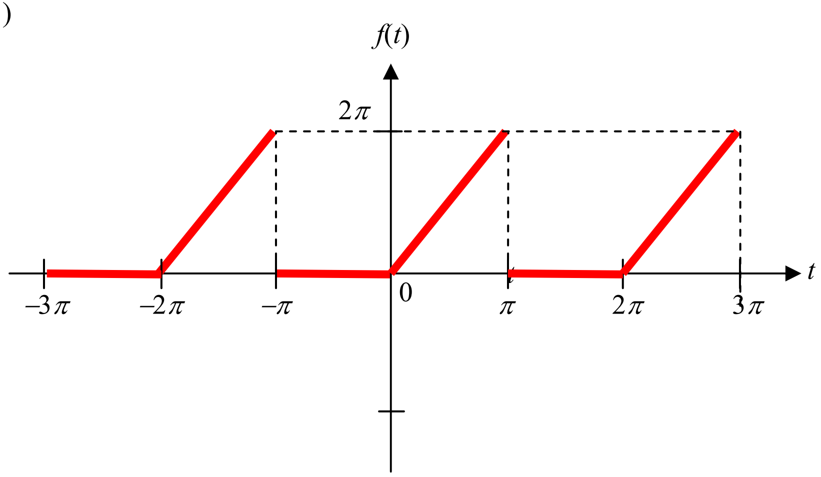
$$f(t)=\cos\left(\omega t+\frac{\pi}{3}\right)+\cos\left(\omega t-\frac{\pi}{3}\right)$$

$$=\mathrm{Re}\left[e^{j\left(\omega t+\frac{\pi}{3}\right)}+e^{j\left(\omega t-\frac{\pi}{3}\right)}\right]=\mathrm{Re}\left[e^{j\omega t}\left(e^{j\frac{\pi}{3}}+e^{-j\frac{\pi}{3}}\right)\right]$$

$$=\mathrm{Re}\left[e^{j\omega t}\cdot 2\cos\frac{\pi}{3}\right]$$

$$=\cos\omega t$$

２．(1)



(2) $a_0=\dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi}f(t)\,dt=\dfrac{1}{\pi}\left(\int_{-\pi}^{0}0\cdot dt+\int_{0}^{\pi}2t\cdot dt\right)=\dfrac{1}{\pi}\left[t^2\right]_0^{\pi}=\pi$

(3)

$$a_n=\frac{1}{\pi}\int_0^{\pi}2t\cos(nt)\,dt=\frac{2}{\pi}\left\{\frac{1}{n}\left[t\sin(nt)\right]_0^{\pi}-\frac{1}{n}\int_0^{\pi}1\cdot\sin(nt)\,dt\right\}$$

$$=-\frac{2}{n\pi}\left[-\frac{1}{n}\cos(nt)\right]_0^{\pi}=\frac{2}{n^2\pi}\left(\cos(n\pi)-\cos 0\right)=2\left\{\frac{(-1)^n-1}{n^2\pi}\right\}$$

(4)

$$b_n=\frac{1}{\pi}\int_0^{\pi}2t\sin(nt)\,dt=\frac{2}{\pi}\left\{-\frac{1}{n}\left[t\cos(nt)\right]_0^{\pi}+\frac{1}{n}\int_0^{\pi}1\cdot\cos(nt)\,dt\right\}$$

$$=\frac{2}{\pi}\left\{\frac{\pi}{n}(-1)^{n+1}+\frac{1}{n^2}\left[\sin(n\pi)\right]_0^{\pi}\right\}=\frac{2(-1)^{n+1}}{n}$$

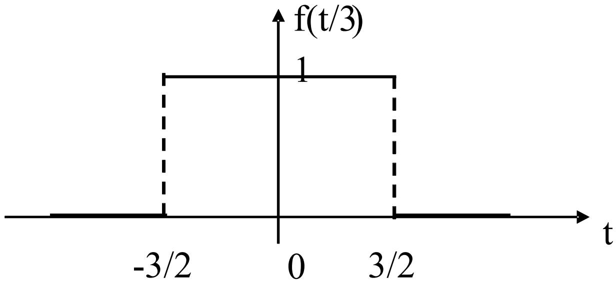
(5)

$$f(t)=\frac{\pi}{2}+2\sum_{n=1}^{\infty}\left\{\frac{(-1)^n-1}{n^2\pi}\cos(nt)+\frac{(-1)^{n+1}}{n}\sin(nt)\right\}$$

$$=\frac{\pi}{2}-\frac{4}{\pi}\left(\cos(t)+\frac{\cos(3t)}{3^2}+\frac{\cos(5t)}{5^2}+\cdots\right)+2\left(\sin(t)-\frac{\sin(2t)}{2}+\frac{\sin(3t)}{3}-\cdots\right)$$

3． $f(t)=\mathrm{rect}(t)$

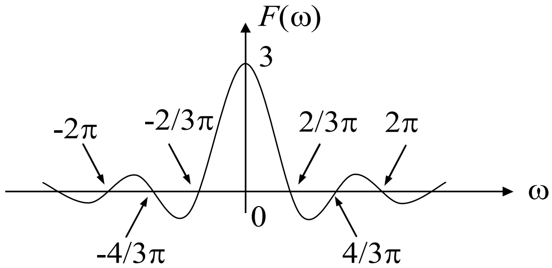
(1)



(2) $F\left[f(t)\right]=\sin c\left(\dfrac{\omega}{2}\right)$, $F\left[f(at)\right]=\dfrac{1}{|a|}F\left(\dfrac{\omega}{a}\right)$ を用いて,

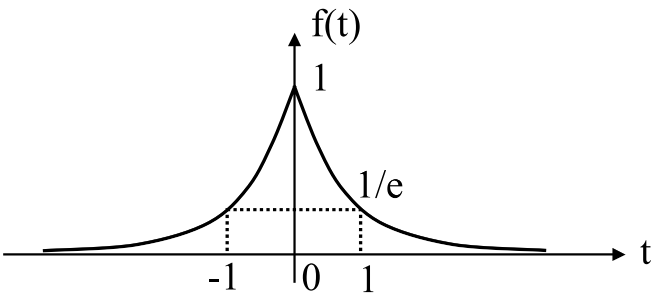
$$F\left[f\left(\frac{t}{3}\right)\right]=3\sin c\left(\frac{3\omega}{2}\right)$$

(3)



4． $f(t)=\exp\left(-|t|\right)$

(1)



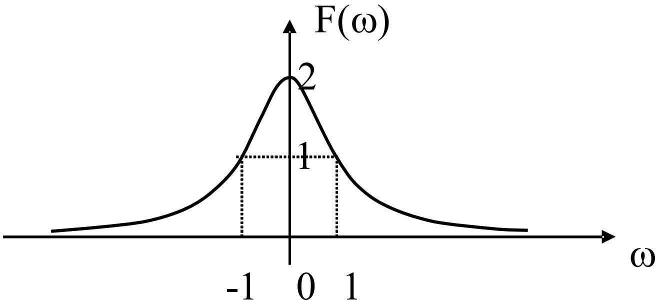
(2) 半値全幅をT_{FWHM}とおく. 振幅の最大値は 1 であるので,

$$\exp\left(-\frac{T_{FWHM}}{2}\right)=\frac{1}{2} \qquad T_{FWHM}=2\ln 2$$

(3)

$$F\left[f(t)\right]=\frac{1}{1-j\omega}+\frac{1}{1+j\omega}=\frac{2}{1+\omega^2}$$

(4)



(5) 半値全幅をF_{FWHM}とおく.

$t=\pm 1$のとき, 振幅の最大値の半分になるので$F_{\mathrm{FWHM}}=2$

5．

(1) $F^{-1}\left[F(3\omega)\right]=\frac{1}{3}f\left(\frac{1}{3}t\right)$

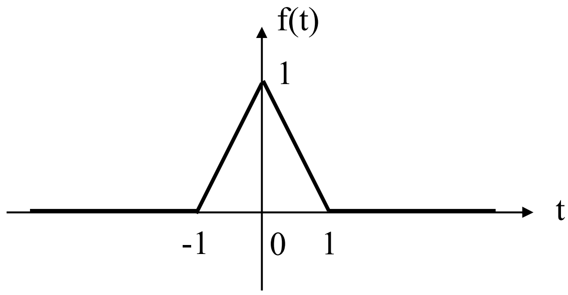
(2) $F^{-1}\left[F(\omega)e^{-j3\omega}\right]=f(t-3)$

(3) $F^{-1}\left[F(\omega)\cos 3\omega\right]=\frac{1}{2}\left[f(t-3)+f(t+3)\right]$

(4) $F^{-1}\left[F(\omega-1)\right]=f(t)e^{jt}$

6． $f(t)=\begin{cases}1-|t| & (|t|<1)\\ 0 & (|t|>1)\end{cases}$

(1)



(2) $f(t)$は実偶関数であることに注目し、

$$\mathcal{F}\left[f(t)\right]=2\int_0^{\infty}f(t)\cos\omega t dt=2\int_0^1(1-t)\cos\omega t dt=2\left[(1-t)\frac{1}{\omega}\sin\omega t\right]_0^1+2\int_0^1\frac{1}{\omega}\sin\omega t dt$$

$$=2\left\{\left[\frac{-1}{\omega^2}\cos\omega t\right]_0^1\right\}=2\left[\frac{1-\cos\omega}{\omega^2}\right]=\frac{2\left(2\sin^2\left(\frac{\omega}{2}\right)\right)}{\omega^2}=\frac{\sin^2\left(\frac{\omega}{2}\right)}{\frac{\omega^2}{4}}=\mathrm{sinc}^2\left(\frac{\omega}{2}\right)$$