フーリエ解析　小テスト３　解答

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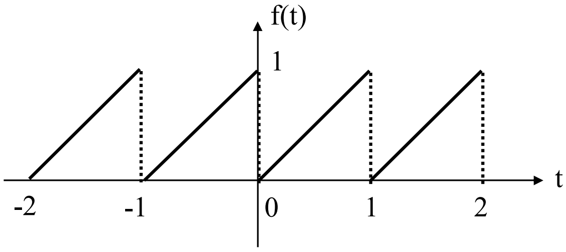
１．$f\left(t\right)=t \quad (0<t<1)$　　ただし、$f\left(t\right)=f\left(t+n\right)$　（n：整数）

(1)



(2)

周期$T=1$　　　　$\therefore \omega_0=\frac{2\pi}{T}=2\pi$

(3)

$$a_0=\frac{2}{T}\int_0^1 t dt=\frac{2}{T}\left[\frac{1}{2}t^2\right]_0^1=1$$

(4)

$$a_n=\frac{2}{T}\int_0^1 t\cos\left(n\omega_0 t\right)dt=\frac{2}{T}\left[t\frac{\sin\left(n\omega_0 t\right)}{n\omega_0}+\frac{\cos\left(n\omega_0 t\right)}{\left(n\omega_0\right)^2}\right]_{\frac{T}{2}}^{T}$$

$$=\frac{2}{T}\left[\left(\frac{\sin\left(n\omega_0\right)}{n\omega_0}+\frac{\cos\left(n\omega_0\right)}{\left(n\omega_0\right)^2}\right)+\left(0+\frac{\cos\left(0\right)}{\left(n\omega_0\right)^2}\right)\right]=0$$

(5)

$$b_n=\frac{2}{T}\int_0^1 t\sin\left(n\omega_0 t\right)dt=\frac{2}{T}\left[-t\frac{\cos\left(n\omega_0 t\right)}{n\omega_0}+\frac{\sin\left(n\omega_0 t\right)}{\left(n\omega_0\right)^2}\right]_0^1$$

$$=\frac{2}{T}\left[\left(-t\frac{\cos\left(n\omega_0\right)}{n\omega_0}+\frac{\sin\left(n\omega_0\right)}{\left(n\omega_0\right)^2}\right)-\left(0+\frac{\sin\left(0\right)}{\left(n\omega_0\right)^2}\right)\right]=-\frac{1}{2}\left(-\frac{1}{2n\pi}\right)=-\frac{1}{n\pi}$$

(6)

$$f\left(t\right)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}\left\{a_n\cos\left(n\omega_0 t\right)+b_n\sin\left(n\omega_0 t\right)\right\}$$

$$=\frac{1}{2}-\frac{1}{\pi}\sum_{n=1}^{\infty}\left\{\frac{\sin\left(2\pi nt\right)}{n}\right\}$$

２．

（１） $\sin^2 \omega_0 t = \frac{1}{2} - \frac{1}{2}\cos 2\omega_0 t$

（２） $\sin^3 \omega_0 t = \frac{3}{4}\sin \omega_0 t - \frac{1}{4}\sin 3\omega_0 t$

（２倍角の公式、３倍角の公式などを使って求める方が簡単）

３．

$f(t) = \frac{t}{T}$　　$(0<t<T)$　　ただし，$f(t) = f(t+T)$

（１）

$$c_0 = \frac{1}{T}\int_0^T \frac{t}{T}dt = \frac{1}{T^2}\left[\frac{1}{2}t^2\right]_0^T = \frac{1}{2} \qquad \omega_0 = \frac{2\pi}{T}$$

（２）

$$c_n = \frac{1}{T}\int_0^T \frac{t}{T}e^{-jn\omega_0 t}dt = \frac{1}{T^2}\left\{\left[t\cdot\frac{1}{-jn\omega_0}e^{-jn\omega_0 t}\right]_0^T - \int_0^T \frac{1}{-jn\omega_0}e^{-jn\omega_0 t}dt\right\}$$

$$= \frac{1}{T^2}\left\{\left[T\cdot\frac{e^{-j2\pi n}}{-jn\omega_0} - 0\right] + \left[\frac{1}{(n\omega_0)^2}e^{-jn\omega_0 t}\right]_0^T\right\} = \frac{1}{T^2}\left\{\left[T\cdot\frac{e^{-j2\pi n}}{-jn\omega_0}\right] + \left[\frac{e^{-j2\pi n}-1}{(n\omega_0)^2}\right]\right\}$$

$$= \frac{1}{T^2}\left\{\left[T\cdot\frac{1}{-jn\omega_0}\right] + \left[\frac{1-1}{(n\omega_0)^2}\right]\right\} = \frac{1}{T}\left[\frac{1}{-jn\omega_0}\right] = j\frac{1}{Tn\omega_0} = \frac{e^{j\frac{\pi}{2}}}{2\pi n}$$