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Estimation of Electric Fields from Magnetic Field Distributions and an Application to Helicon Wave

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General formulae of electric fields of the arbitrarily excited waves are derived from magnetic field distributions in a cylindrical cold plasma for experimental use. As an example of an application, explicit expressions of the electric fields, including wave energy density and energy flux of the helicon wave, are presented.

KEYWORDS: plasma, electric field, magnetic field, wave energy density, energy flux, Poynting vector, helicon wave, ICP, dusty plasma

A great variety of waves can be excited in a plasma and wave phenomena can be categorized in many ways—as electrostatic or electromagnetic waves; waves propagating parallel or perpendicular to a static magnetic field; wave frequency and wavelength compared with a characteristic frequency and scale length, respectively—and also by various boundary conditions (see also the Clemmow-Mullaly-Allis (CMA) diagram).[1] Although wave magnetic fields $\boldsymbol{B}$ in relatively low-temperature plasmas can be easily and directly measured, using magnetic probes for instance, excited electric fields $\boldsymbol{E}$ with time-varying components (more than MHz frequency range), which contain inductive (electromagnetic) as well as space charge (electrostatic) terms, are much more difficult to measure experimentally.[2] Since poor knowledge of the electric fields $\boldsymbol{E}$ results in a correspondingly poor understanding of wave natures, reliable methods to estimate the $\boldsymbol{E}$ fields are of crucial importance. If the magnetic and electric fields are known, wave characteristics such as excited wave structures, absolute values of wave energy and energy flux can be obtained.

In this letter, we propose a method of deriving general formulae of $\boldsymbol{E}$ (electromagnetic component) from known data of $\boldsymbol{B}$ in a cylindrical plasma. Since only the cold plasma dispersion relation and Maxwell's equations are used in this calculation, the obtained result is a general one for arbitrary excited waves. As an example of one application, explicit expressions of $\boldsymbol{E}$ (from $\boldsymbol{B}$), wave energy density W and wave energy flux $\boldsymbol{S}$ are presented for the helicon wave[3–8] case. The use of helicon waves to produce a high-density-plasma source has become very attractive in confinement devices as well as in plasma processing of materials, and detailed knowledge of the nature of waves from the viewpoint of the plasma production mechanism, including plasma initiation, is essential.

First, we will derive general formulae for the excited electric fields. Using international system (SI) mks units, a cold plasma dielectric tensor K can be represented by[1]

$$\mathsf{K} = \begin{pmatrix} K_\perp & -iK_\times & 0 \\ iK_\times & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{pmatrix}, \tag{1}$$

where each element is defined as

$$K_\parallel = 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2}, \quad K_\perp = 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2 - \omega_{cj}^2},$$
$$K_\times = \sum_j \frac{\omega_{pj}^2}{\omega^2 - \omega_{cj}^2} \cdot \frac{\varepsilon_j \omega_{cj}}{\omega},$$
$$\omega_{pj}^2 = \frac{n_j q_j^2}{m_j \varepsilon_0}, \quad \omega_{cj} = \frac{|\, q_j \,| B_0}{m_j}. \tag{2}$$

In addition, the following notations are used: ω, wave angular frequency; ε_0, dielectric constant in the vacuum; m_j, mass; n_j, number density; q_j, charge; ε_j, sign of the charge; B_0, static magnetic field along the z direction.

Using time-varying electric and magnetic fields of $e^{-i\omega t}$ and Maxwell's equation of $\nabla \times \boldsymbol{B} = \mu_0(\partial \boldsymbol{D}/\partial t)$ (μ_0: permeability in the vacuum, $\boldsymbol{D}$: electric displacement) in the cylindrical (r, θ, z) coordinate, the electric fields are written in terms of the magnetic fields as

$$E_r = \frac{ic^2}{\omega(K_\perp^2 - K_\times^2)} \left[K_\perp \left(\frac{1}{r}\frac{\partial B_z}{\partial \theta} - \frac{\partial B_\theta}{\partial z} \right) + iK_\times \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) \right],$$
$$E_\theta = \frac{-ic^2}{\omega(K_\perp^2 - K_\times^2)} \left[iK_\times \left(\frac{1}{r}\frac{\partial B_z}{\partial \theta} - \frac{\partial B_\theta}{\partial z} \right) - K_\perp \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) \right],$$
$$E_z = \frac{ic^2}{\omega K_\parallel} \left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) - \frac{1}{r}\frac{\partial B_r}{\partial \theta} \right], \tag{3}$$

where c is a velocity of light. Even though the electric fields are strongly affected by the boundary conditions in some cases, eq. (3) is still valid, since the magnetic fields obtained inherently satisfy these conditions. When the electric fields are measured independently by some means, these estimated electric fields can be used for checking the reliability of the experimental data.

The wave energy density W is defined in two ways[1,9] as

$$W = \frac{1}{2}\mathrm{Re}\left[\frac{\boldsymbol{B}^* \cdot \boldsymbol{B}}{2\mu_0} + \frac{\varepsilon_0}{2}\boldsymbol{E}^* \cdot \frac{\partial}{\partial \omega}(\omega \mathsf{K}_h) \cdot \boldsymbol{E} \right]$$
$$= \frac{1}{2}\mathrm{Re}\left[\frac{\varepsilon_0}{2}\boldsymbol{E}^* \cdot \frac{\partial}{\omega \partial \omega}(\omega^2 \mathsf{K}_h) \cdot \boldsymbol{E} \right], \tag{4}$$

where the asterisk, Re and K_h indicate a complex conjugate, real part and Hermitian part of the dielectric

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tensor, respectively.

The energy flux $\boldsymbol{S}$, composed of the Poynting vector $\boldsymbol{P}$ and the nonelectromagnetic energy flux $\boldsymbol{T}$ due to coherent particle motions, is given by the following equations.[1)]

$$\boldsymbol{S}=\boldsymbol{P}+\boldsymbol{T},$$
$$\boldsymbol{P}=\frac{1}{2\mu_0}\,\mathrm{Re}\,(\boldsymbol{E}^*\times\boldsymbol{B}),$$
$$\boldsymbol{T}=-\frac{\omega\varepsilon_0}{4}\boldsymbol{E}^*\cdot\frac{\partial}{\partial\boldsymbol{k}}\mathsf{K}_h\cdot\boldsymbol{E}. \qquad (5)$$

Therefore, the energy density W and energy flux $\boldsymbol{S}$ can be estimated generally by using magnetic fields data and electric fields derived from eq. (3).

Next we will apply this method to the helicon wave, in which $\boldsymbol{B}$ and $\boldsymbol{E}$ are proportional to $e^{i(k_z z+m\theta-\omega t)}$ (k_z: parallel wave number, m: azimuthal mode number).[4)] Assuming $\omega_{ci}\ll\omega\ll\omega_{ce}$, $\omega\ll\omega_{pe}$ and $\omega\omega_{ce}\ll\omega_{pe}^2$ ($m_e\ll m_i$, composition of electrons and one species of ions) from eq. (3), the $\boldsymbol{E}$ fields become

$$E_r=\omega_{ce}\left(\frac{c}{\omega_{pe}}\right)^2\left(ik_zB_r-\frac{\partial B_z}{\partial r}\right),$$
$$E_\theta=\omega_{ce}\left(\frac{c}{\omega_{pe}}\right)^2\left(ik_zB_\theta-\frac{im}{r}B_z\right),$$
$$E_z=-\omega\left(\frac{c}{\omega_{pe}}\right)^2\left[\frac{m}{r}B_r+i\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)\right]. \qquad (6)$$

From these equations, it can be seen that E_z can be neglected when $\omega/2\pi$ is several MHz and B_0 is larger than several 10^{-3} T since $E_r\sim E_\theta\sim(\omega_{ce}/\omega)E_z$, as long as no wave damping occurs. If there is a flat density profile in this (no damping) case, the helicon wave fields are written as[4)]

$$B_r=C_1J_{m-1}+C_2J_{m+1},$$
$$B_\theta=i(C_1J_{m-1}-C_2J_{m+1}),$$
$$B_z=C_3J_m, \qquad (7)$$

and

$$E_r=(\omega/k_z)B_\theta,$$
$$E_\theta=-(\omega/k_z)B_r,$$
$$E_z=0, \qquad (8)$$

where C_1, C_2 and C_3 are as defined in ref. 4 and $J_m(k_\perp r)$ is a Bessel function ($k_\perp$: transverse wave number).

Under typical experimental conditions, however, the collision frequency ν becomes large ($\nu/\omega>0.1$ and the observed E_z is not neglected[10)]) when the electron density n_e is more than about 10^{11} cm^{-3}, the electron temperature T_e is several eV and the wave frequency $\omega/2\pi$ is on the order of several MHz. Since collisional damping cannot be neglected in this case, m_e in ω_{pe} and ω_{ce} in eq. (2) must be replaced by $(1+i\nu/\omega)m_e$ to give k_z+ik_d (k_d: damping wave number along the z axis). In addition, for the case when electron Landau damping becomes large, ν_{LD} in $K_\parallel$ is needed to give $\nu_{LD}+\nu$, where $\nu_{LD}=2\pi^{1/2}\omega(\omega/k_zv_{te})^3\exp(-\omega^2/k_z^2v_{te}^2)$ and v_{te} is the electron thermal velocity.[4)] Note that for this damping case, E_r and E_θ in eq. (6) remain the same, while ω must be replaced by $\omega+i(\nu+\nu_{LD})$ in E_z.

For the helicon wave, the boundaries are highly important and may force the plasma to generate a relatively large electrostatic field, which causes some difficulty in deriving $\boldsymbol{E}$ from $\boldsymbol{B}$ fields because the $\boldsymbol{B}$ fields become small. It is considered for the low wave damping case that the ratio of electrostatic to electromagnetic components is high ($\sim k_\perp/k_z$) near the plasma surface, becomes nearly one at the plasma center and takes a very small value ($\ll 1$) at the intermediate region, for $m=1$ and $m=-1$ excitation. However, this ratio may be large (on the order of $k_\perp/k_z$) over the whole plasma region for $m=0$ excitation. According to our preliminary measurements ($k_\perp/k_z=$ 3–7), there does not seem to be a problem when deriving $\boldsymbol{E}$ from the $\boldsymbol{B}$ fields at the inner plasma region; *e.g.*, the perpendicular electric field (23 cm axially away from the antenna edge) is several V/cm for $m=1$ and less than one V/cm for $m=-1$ excitation. The experimentally obtained electrostatic component suggests small at the inner plasma region compared with that near the plasma surface.[10)] Of course, generally, the above derivation using $\nabla\times\boldsymbol{B}=\mu_0(\partial\boldsymbol{D}/\partial t)$ cannot be applied without large error if the electrostatic condition of $|K_{ij}|\ll|c/(\omega/k)|^2$ (for all ij component) is satisfied[1)] (note that typically $K_{zz}\gg|c/(\omega/k)|^2$ for the helicon wave).

The $\boldsymbol{E}$ fields for the case of no magnetic field are

$$E_r=\frac{ic^2}{\omega K_\parallel}\left(\frac{1}{r}\frac{\partial B_z}{\partial\theta}-\frac{\partial B_\theta}{\partial z}\right),$$
$$E_\theta=\frac{ic^2}{\omega K_\parallel}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right),$$
$$E_z=\frac{ic^2}{\omega K_\parallel}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)-\frac{1}{r}\frac{\partial B_r}{\partial\theta}\right], \qquad (9)$$

where $K_\parallel=1-(\omega_{pe}/\omega)^2$ as $m_e\ll m_i$, and this becomes $-(\omega_{pe}/\omega)^2$ for a higher density plasma in which $\omega\ll\omega_{pe}$. This condition may correspond to inductively coupled plasma (ICP).[11)] The same equations may also be used for the dusty plasma[12)] ($\omega_{pd}\ll\omega_{pi}$, $\omega_{pi}\ll\omega_{pe}$ and $\omega_{cd}\ll\omega_{ci}\ll\omega_{ce}$, where d denotes the dust particles), if the magnetic field $B=0$. For the extreme case that n_e is low enough to give $\omega_{pe}\ll\omega_{pi}$, $K_\parallel$ becomes $1-(\omega_{pi}/\omega)^2$. Note that these E fields express only a wave propagating component, not an evanescent one.

For the case of a weak static magnetic field ($\omega_{ce}\ll\omega$), the $\boldsymbol{E}$ fields are derived from eq. (3) with $K_\parallel=K_\perp=1-(\omega_{pe}/\omega)^2$ and $K_\times=-(\omega_{pe}/\omega)^2(\omega_{ce}/\omega)$, and for the strong field case ($\omega\ll\omega_{ci}\ll\omega_{ce}$)$K_\parallel=1-(\omega_{pe}/\omega)^2$, $K_\perp=1+(\omega_{pi}/\omega_{ci})^2$ and $K_\times=-(\omega_{pi}/\omega_{ci})^2(\omega/\omega_{ci})$.

The xx ($=yy$) and zz components of $\partial(\omega\mathsf{K}_h)/\partial\omega$ without an approximation are $1+\omega_{pe}^2(\omega^2+\omega_{ce}^2)/(\omega^2-\omega_{ce}^2)^2+\omega_{pi}^2(\omega^2+\omega_{ci}^2)/(\omega^2-\omega_{ci}^2)^2$ and $1+\omega_{pe}^2/\omega^2+\omega_{pi}^2/\omega^2$, respectively, and also, those of $\partial(\omega^2\mathsf{K}_h)/\omega\partial\omega$ are $2+2\omega_{pe}^2\omega_{ce}^2/(\omega^2-\omega_{ce}^2)^2+2\omega_{pi}^2\omega_{ci}^2/(\omega^2-\omega_{ci}^2)^2$ and 2, respectively. From these components under the conditions of $\omega_{ci}\ll\omega\ll\omega_{ce}$ (helicon wave) and $m_e\ll m_i$, the energy density W is given by

$$W = \frac{1}{2}\,\mathrm{Re}\left\{\frac{\boldsymbol{B}^*\cdot\boldsymbol{B}}{2\mu_0} + \frac{\varepsilon_0}{2}\left[\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}+\frac{\omega_{pi}^2}{\omega^2}\right)\times(E_r^*E_r + E_\theta^*E_\theta) + \left(1+\frac{\omega_{pe}^2}{\omega^2}\right)E_z^*E_z\right]\right\}$$
$$= \frac{\varepsilon_0}{2}\,\mathrm{Re}\left[\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}\right)(E_r^*E_r + E_\theta^*E_\theta) + E_z^*E_z\right]. \qquad (10)$$

Substituting for eq. (6) into eq. (10), W is explicitly determined from the magnetic field data. For the case of no magnetic field, $1+\omega_{pe}^2/\omega_{ce}^2+\omega_{pi}^2/\omega^2$ in the first equation of eq. (10) must be replaced by $1+\omega_{pe}^2/\omega^2$, and in the second equation, $1+\omega_{pe}^2/\omega_{ce}^2$ must be replaced by 1.

Now we can consider the energy flux $\boldsymbol{S}$ of the helicon wave. The nonelectromagnetic energy flux $\boldsymbol{T}$ is zero because K_h is independent of $\boldsymbol{k}$. Hence, each component of $\boldsymbol{S}$ $(=\boldsymbol{P})$ is given explicitly by the magnetic field, described by eqs. (5) and (6). Note that the energy flux estimation from magnetic fields is correct even though there exist electrostatic fields. As one example, the z component of $\boldsymbol{S}$ (along the static magnetic field) is given by

$$S_z = \frac{-\omega_{ce}}{2\mu_0}\left(\frac{c}{\omega_{pe}}\right)^2 \mathrm{Re}\left(\frac{\partial B_z^*}{\partial r}B_\theta + \frac{im}{r}B_rB_z^*\right). \qquad (11)$$

(For the case of no magnetic field, $\boldsymbol{T}$ is again zero and $\boldsymbol{S}$ $(=\boldsymbol{P})$ is determined from eqs. (5) and (9).) If we can use eqs. (7) and (8), S_z for the helicon wave becomes

$$S_z = \frac{1}{2\mu_0}\left(\frac{\omega}{k_z}\right)\mathrm{Re}\,(B_r^*B_r + B_\theta^*B_\theta). \qquad (12)$$

Preliminary experiments have shown the possibility of estimating the S_z value at the plasma center by the use of eq. (11), from the radial magnetic field distribution and the central electron density. Obtained values for both $m=1$ and $m=-1$ excitation do not differ from the ones using eq. (12) (from the perpendicular component of the magnetic field at the plasma center) within a factor of 1.5.

For realistic analysis of the experimental data, eq. (6) can be very useful instead of eqs. (7) and (8) which have been used for approximate distributions so far.[3,6,7] For example, deriving S_z or W along the z axis from eqs. (10) and (11) gives us information about the energy flow and absolute wave damping, as discussed above. If the excited wave data contain many m mode numbers and/or many k_z values (this needs a complicated treatment of decomposing m and k_z spectra), eqs. (3)–(5) should be used instead.

General formulae of the excited electric fields, which are difficult to measure in most cases, have been estimated from magnetic field distributions in cylindrical cold plasma. To show an application of these formulae, explicit expressions of the electric fields $\boldsymbol{E}$ including the wave energy density W and the energy flux $\boldsymbol{S}$ of the helicon wave have been presented.

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