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Excitation of the Second Harmonics Due to a Resonant Mode-Mode Coupling for the Ion Bernstein Wave in Tokamaks

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The excitation of the second harmonics and its wave damping near the subharmonic cyclotron resonance layer of $\omega=2.5\omega_D$ have been proposed for the ion Bernstein wave in tokamaks (ω_D: deuteron cyclotron frequency). By the use of three-dimensional ray tracing and a coupling theory, it is found that the second harmonic wave can be excited as a real wave through a nonlinear process (resonant mode-mode coupling). This excited wave can propagate toward the high-field side, and lose its energy through harmonic cyclotron damping and/or electron Landau damping.

In recent years, a directly launched ion Bernstein wave (IBW)[1] heating has been a promising method of additional heating. The first experiments of this wave were carried out in an ACT-1 toroidal device.[2,3] In tokamaks, experiments were conducted in JIPPTII-U,[4,5] PLT,[6,7] TNT-A[8-10] and Alcator C.[11,12]

One of the most interesting results in recent tokamak experiments is a subharmonic cyclotron resonance heating by the application of the IBW: the hydrogen ion damping at $\omega_0=1.5\omega_H$ (JIPPTII-U), deuterium ion damping at $\omega_0=1.5\omega_D$ (PLT), ion heating at $\omega_0=2.5\omega_D$ (TNT-A) and ion heating at $\omega_0=1.25$, 1.5 and $2.5\omega_H$ (Alcator C). Here, ω_D and ω_H are the deuteron and proton cyclotron frequencies, respectively.

Theoretically, a nonlinear ion heating at the subharmonic cyclotron frequency of the IBW can be expected. This ion heating results from enhancement of acceleration due to a particle trapping,[13] or nonlinear ion cyclotron Landau damping of the virtual wave due to self-interaction (nonresonant wave-wave scattering) of the IBW.[14]

Here, we propose another heating scheme due to a resonant mode-mode coupling; an excitation of the second harmonics (SH) of the launched wave (LW) as a real wave due to nonlinearity (self-interaction), and the linear ion cyclotron damping at $2\omega_0=n\omega_{ci}$ (n: integer, ω_{ci}: ion cyclotron frequency) and/or electron Landau damping near the subharmonic resonance layer ($\omega_0=(n/2)\omega_{ci}$). This scheme may be applied to an overdensity plasma (electron heating over the cutoff plasma density).

In this letter, a ray trajectory and its wave damping are calculated by the use of the ray tracing code.[15] We apply this code to the PLT tokamak as an example in order to derive the conditions required to excite the SH. The coupling of the LW and the excited SH is also estimated.

The conditions for exciting the real SH wave due to self-interaction are that the frequency and wave number of the SH are twice as large as those of the LW (phase matching condition), and that the coupling of two waves is strong. We have investigated the former conditions of the matching by the ray trace,[16] modifying the code[15] provided by M. Ono *et al.*[17] The latter condition of the coupling is described later.

We have studied the LW propagation with a frequency ω_0, and parallel (along the magnetic field) and perpendicular wave numbers, k_{z0} and k_{x0}, respectively, which change along the trajectory. We derive the perpendicular wave number k_{x1} for the SH from the dispersion relation $D(k_x, k_z, \omega)=0$, using the frequency

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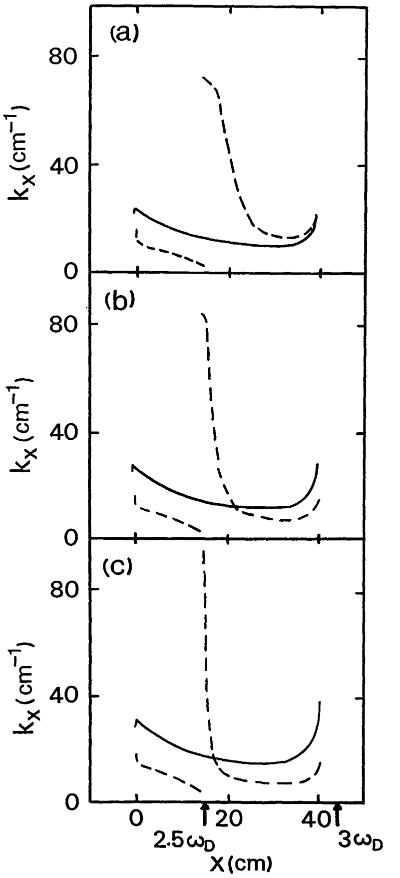

Fig. 1. Radial profiles of the perpendicular wave number k_x for different concentration of $n_H/(n_H+n_D)=0.1$ (case (a)), 0.5 (case (b)) and 0.8 (case (c)) in PLT (n_H and n_D are hydrogen and deuterium densities, respectively). Here, the solid and broken lines show the launched wave (LW) of k_{x0} and excited SH wave of $k_{x1}/2$, respectively. The initial parallel refractive index n_{z0} and thus the wave number k_{z0} are 2.0 and 0.0201 cm^{-1}, respectively.

$\omega_1=2\omega_0$ and parallel wave number $k_{z1}=2k_{z0}$ on the trajectory. At the point where $k_{x1}=2k_{x0}$ is satisfied, i.e., $D(k_{x0}, k_{z0}, \omega_0)=D(k_{x1}, k_{z1}, \omega_1)=0$, excitation of the SH as a real wave can be expected due to the matching condition.

Figure 1 shows an example of radial profiles of k_{x0} and k_{x1} for the different initial wave number k_{z0} with two species plasma in the PLT tokamak (nearly the same conditions of ref. 6 except for the frequency and concentration ratio). The parameters used for the calculations are $f=48$ MHz, $B_t=2.8$ T, $T_e(0)=1300$ eV, $T_i(0)=800$ eV, $n_e(0)=2.5\times10^{13}$ cm^{-3} and $\omega_0\lesssim3\omega_D$ at the plasma surface. For the case of a low concentration of the hydrogen (Fig. 1(a)), k_{x1} is twice as large as k_{x0} near the plasma edge. With an increase in this hydrogen concentration, i.e., from Figs. 1(a) to 1(c), the point of the excitation of the SH approaches the subharmonic resonance layer (when the hydrogen density is lower than $\sim10\%$ of the deuterium density, a generation of the SH near the plasma edge cannot occur as $k_{x1}>2k_{x0}$). This SH wave, excited near the plasma edge (low-hydrogen concentration case) or before the subharmonic resonance layer (higher-hydrogen concentration case), propagates towards the plasma core. Then, it damps strongly at the subharmonic resonance layer, because almost 100% single path absorption is expected on the layer of $2\omega_0=5\omega_D$ (in this case electron Landau damping is small). Similar numerical results are found in JIPPTII-U and TNT-A with $2.5\omega_D$ heatings; excitation of the SH occurs near the plasma edge for the case in which the hydrogen concentration ratio is about 10–20%.

As for the subharmonic heating with single species plasmas such as $\omega_0=1.5\omega_H$, $2.5\omega_H$ and $1.5\omega_D$, we have also found the excitation of the SH and its wave damping near the plasma center in PLT and JIPPTII-U, TNT-A and Alcator C devices. In this case, $k_{x1}>2k_{x0}$ is satisfied in the region from the plasma edge to the subharmonic resonance layer. The SH can be generated just beyond the subharmonic resonance layer and lose its energy due to the harmonic cyclotron damping and/or electron Landau damping near the plasma center. With an increase in the initial parallel wave number k_{z0}, the generating point of the SH approaches the subharmonic resonance layer. As our assumption of a weakly damped wave breaks down near the subharmonic resonance layer in this case, there is a possibility that the SH, which is generated near this layer, is evanescent to start with.

So far, we have proposed $2.5\omega_D$ heating and have also found that the matching condition is satisfied in the present experiments. Hereafter, we will derive the coupling coefficient of the LW and SH. The basic equations of the distribution function and the current perturbation for the nonlinear response were derived in eqs. (97) and (99) of ref. 18, respectively, by the use of the Vlasov equation. Under the assumptions of (i) Maxwell distribution for the unperturbed equilibrium distribution, (ii) electrostatic approximation (k_{x0} and electric field E_{x0} survive) and (iii) self-interaction of two waves (the wave number k_{x0} and frequency ω_0 are the same for two waves), the current perturbation J_{x1} (the oscillating part is neglected) is obtained in the following form (CGS Unit):

$$J_{x1}=aE_{x0}^2\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\int_0^{\infty}\sqrt{\mu}\,(J_{n-1}(2b\sqrt{\mu})+J_{n+1}(2b\sqrt{\mu}))g_{n,l}\,d\mu, \tag{1}$$

where

$$g_{n,l}=\frac{2\exp(-\omega_{ci}\mu/T_i)}{(2\omega_0-n\omega_{ci})(\omega_0-l\omega_{ci})}\left[2(J_{n-l+1}(b\sqrt{\mu})+J_{n-l-1}(b\sqrt{\mu}))\int_{-\infty}^{\infty}A_l\,dp_z + (J_{n-l+1}(b\sqrt{\mu})-J_{n-l-1}(b\sqrt{\mu}))\,l\int_{-\infty}^{\infty}B_l\,dp_z\right], \tag{2}$$

$$\int_{-\infty}^{\infty}A_n\,dp_z=[(-\omega_{ci}\mu/T_i)+1/2](J_{n-1}(b\sqrt{\mu})+J_{n+1}(b\sqrt{\mu})) -(k_{x0}/2)\sqrt{1/2m_i\omega_{ci}}\sqrt{\mu}\,(J_{n+2}(b\sqrt{\mu})-J_{n-2}(b\sqrt{\mu})), \tag{3}$$

$$\int_{-\infty}^{\infty}B_n\,dp_z=J_{n-1}(b\sqrt{\mu})+J_{n+1}(b\sqrt{\mu}), \tag{4}$$

$$a=(n_0e^3/8T_i^2)\sqrt{\omega_{ci}^3/2m_i^3}, \tag{5}$$

$$b=\sqrt{2k_{x0}^2/m_i\omega_{ci}}, \tag{6}$$

and μ is the modified magnetic moment (the perpendicular particle energy divided by ω_{ci}). Here, ω_{ci}, T_i, n_0, $-e$ and m_i are ion cyclotron frequency, ion temperature, plasma density, electronic charge and atomic mass, respectively.

The excited electric field E_{x1} of the SH is derived from J_{x1} and the Maxwell equations. The threshold power P_{th}, emitting from the antenna, for the effective nonlinear excitation of the SH is defined so that a coupling coefficient $C\equiv E_{x1}/E_{x0}$ is 0.5. As can be seen from eq. (5), the coupling becomes stronger with an increase in the plasma density and/or a decrease in the ion temperature, and stronger near the ion cyclotron harmonic or subharmonic resonance layers from eq. (2).

Now, we will apply these formulae to the present devices. Since the ray trajectory depends on the antenna shape (finite lengths along the toroidal and poloidal directions) and the initial wave numbers, there is a spread of the ray as the wave propagates. Therefore, the threshold value of P_{th} described below is a rough evaluation due to the difficulty of an exact estimation. For the above-mentioned case of PLT with $\omega_0=2.5\omega_D$, P_{th} is less than 1 kW for both cases of Figs. 1(a) and 1(c), since the generating points of the SH are near the $3\omega_D$ and $2.5\omega_D$ resonance layers, respectively, where the coupling is large (for the case of Fig. 1(a), the spread of the ray in the neighborhood of the antenna is small, which lowers the P_{th} value). For the case of Fig. 1(b), P_{th} is ~ 10 kW as the generating point is located in the middle of the subharmonic and harmonic resonance layers and E_{x0} is relatively small. For the case of PLT with $1.5\omega_D$ heating (ref. 6), P_{th} is ~ 20 kW. In JIPPTII-U with $1.5\omega_H$ (ref. 5) heating, P_{th} is an order of 10 kW. These values are far below the experimental input power.

In conclusion, the SH wave generation and plasma heating near the ion cyclotron subharmonic resonance layer have been proposed in tokamaks by the use of three-dimensional ray tracing for the ion Bernstein wave at $\omega_0=2.5\omega_D$. As the ray propagates from the low-field side, the ion Bernstein wave with twice the frequency and wave number of the launched wave can be excited as a real wave through the nonlinear process and lose its energy. This nonlinear process may explain the experimental results. The coupling of the LW and SH is calculated, and the threshold power P_{th} for the SH wave excitation is smaller than the experimental input power.

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References

1) I. B. Bernstein: Phys. Rev. **109** (1958) 10.
2) M. Ono, K. L. Wong and G. A. Wurden: Phys. Fluids **26** (1983) 298.
3) M. Ono, G. A. Wurden and K. L. Wong: Phys. Rev. Lett. **52** (1984) 37.
4) K. Toi, T. Watari, K. Ohkubo, K. Kawahata, N. Noda, *et al.*: in *Plasma Physics and Controlled Nuclear Fusion Research, 1984* (*Proc. 10th Int. Conf. London, 1984*) IAEA, Vienna (1985) Vol. 1, p. 523.
5) M. Ono, T. Watari, R. Ando, J. Fujita, Y. Hirokura, *et al.*: Phys. Rev. Lett. **54** (1985) 2339.
6) J. Hosea, R. Bell, A. Cavallo, S. Cohen, P. Colestock, *et al.*: in *Proc. 12th European Conf. on Controlled Fusion and Plasma Physics*, (1985) Vol. 2, p. 120.
7) M. Ono, P. Beiersdorfer, R. Bell, S. Bernabei, A. Cavallo, *et al.*: in *Plasma Physics and Controlled Nuclear Fusion Research, 1986* (*Proc. 11th Int. Conf. Kyoto, 1986*) IAEA, Vienna (1987) Vol. 1, p. 477.
8) S. Shinohara, O. Naito and K. Miyamoto: Nucl. Fusion **26** (1986) 1097.
9) S. Shinohara, O. Naito, Y. Ueda, H. Toyama and K. Miyamoto: J. Phys. Soc. Jpn. **55** (1986) 2648.
10) O. Naito, S. Shinohara and K. Miyamoto: J. Phys. Soc. Jpn. **56** (1987) 2988.
11) M. Porkolab, P. Bonoli, K. I. Chen, C. Fiore, R. Granetz, *et al.*, in *Plasma Physics and Controlled Nuclear Fusion Research, 1986* (*Proc. 11th Int. Conf. Kyoto, 1986*) IAEA, Vienna (1987) Vol. 1, p. 509.
12) Y. Takase, J. D. Moody, C. L. Fiore, F. S. McDermott, M. Porkolab and J. Squire: Phys. Rev. Lett. **59** (1987) 1201.
13) H. Abe, H. Okada, R. Itatani and H. Okuda: Phys. Rev. Lett. **53** (1984) 1153.
14) M. Porkolab: Phys. Rev. Lett. **54** (1985) 434.
15) S. Shinohara, O. Naito, K. Ida and K. Miyamoto: Jpn. J. Appl. Phys. **26** (1987) 505.
16) T. H. Six: *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).
17) M. Ono, T. H. Stix, K. L. Wong and R. Horton: in *Proc. 2nd Joint Grenoble-Varenna Int. Symp. on Heating in Toroidal Plasma* (1980) Vol. 1, p. 593.
18) C. S. Liu and V. K. Tripathi: Phys. Rep. **130** (1986) 143.