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Passive Feedback Control of Plasma Position in a Tokamak

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The effect of many passive coils on the positional instabilities in tokamaks is numerically investigated in the case of $t \ll \tau_e$ (effective skin time of the coils), assuming the rigid shift of the plasma column. In order to make the positional stability region of the decay index broader, the passive coils must be located as close as to the plasma surface with the optimum position. The skin time of the coils is determined mainly by the number, cross section area and the conductivity of the coils. Application of the analysis to two small tokamaks shows the fairly good agreements with the experimental results.

§1. Introduction

It is well known that a vertical magnetic field is necessary to maintain the plasma equilibrium in a tokamak. The stability condition of the curvature of the vertical magnetic field is expressed as $1<n<\frac{3}{2}$, where n is the decay index $(=-(R/B_z)(\partial B_z/\partial R))$.[1,2] If that condition is not satisfied, the equilibrium is unstable and there appears the positional instabilities, *e.g.* a vertical or horizontal displacement of the plasma column.

In order to raise the beta value (ratio of the plasma pressure to the magnetic pressure), we must make the vertically elongated cross section plasma,[3-5] but there appears the positional instability (vertical) when $n<0$. Although the local stability criteria suggest the use of the horizontally elongated cross section plasma of droplet shapes,[6] the positional instability (horizontal) occurs when n becomes large.

These instabilities can be stabilized by the shell with infinite conductivity theoretically,[7-10] or by the set of conductors with finite conductivity if the plasma duration is so short that they are regarded as perfect conductors.[11,12] However, when the plasma life time becomes long in large tokamaks, the shell or conductors with finite conductivity cannot stabilize the positional instabilities,[13] but reduce the growth rates.[10,14,15] In large fusion devices, it is important to use both the passive coils to reduce the growth rates and the active feedback coils[16,17] to control the unstable slow motions.

The advantages of the passive coils instead of the shell are, (i) small area, (ii) easiness to mount the coils, (iii) easiness to connect them against the positional instabillity, (iv) possibility of use of coils together with the active feedback control.

The vertical stability of elongated tokamak plasma by the energy principle has been calculated, assuming a rigid shift.[18] The plasma current distribution is replaced by many toroidal filaments and a set of discrete conductors outside the plasma and a virtual shell are used. In this paper, the effect of the passive coils on the positional instability is investigated by estimation of the destabilizing force and the stabilizing force, which is derived from the induced currents by the shift of the plasma column. This calculation is quite easy because of not using an energy principle, and the result of the stability region of the decay index is useful in the case of designing a non-circular cross section tokamak.

In §2, the formulation of the critical n index (stability boundary), the growth rate of the positional instabilities and the effective skin time of the coils are derived assuming the rigid shift, which is the most unstable motion for the elliptic cross section.[19] The positional instabilities can be stabilized if the n index is within the critical decay index, and the plasma duration is too short compared with the skin time of the coils. In §3, the application to some tokamak devices and the optimum position of the coils are presented. In §4, the conclusion is presented.

§2. Calculation of the Shell Effect

2.1 *Vertical displacement of the plasma column*

We consider the configuration that a tokamak plasma is surrounded by $2N_p$ passive coils. In simplicity, circular cross section plasma is assumed to calculate the effect of passive coils in this section. The formulation of critical decay index, growth rate and the effective skin time, derived below, can be easily modified in the case of non-circular cross section plasma.

The flux at i-th coil is represented as

$$\phi_i=\sum_{j=1}^{2N_p} M_{ij}I_j+M_{pi}I_p, \tag{1}$$

where M_{ij} is the mutual inductance between the i and j-th coil, and $M_{ii}=L_i$, I_i are the self inductance, induced current in the i-th coil, respectively. The mutual inductance between the plasma and the i-th coil is expressed as M_{pi} and the plasma current as I_p. We consider here that the plasma column displaces vertically as a rigid body and the passive coils are placed to act as a shell against the vertical displacement of the plasma, and I_p does not change from the poloidal flux conservation. Hereafter we use the cylindrical coordinate (r, ϕ, z) $(r, \phi, z$: distance to the axis, azimuthal angle, distance from the equatorial plane). When each of two coils, symmetrically located at $z=0$ plane (Fig. 1(a) is an example in the case of $N_p=3$), are connected in parallel, and the effects of other coils, *e.g.* the active coils, can be neglected, the differential equation is derived from eq. (1),

$$\mathbf{A}\cdot\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}+\mathbf{B}\cdot\boldsymbol{x}=\boldsymbol{y}, \tag{2}$$

where

$$A_{ij}=M_{2i-1,2j-1}-M_{2i-1,2j},$$

$$B_{ij}=\delta_{ij}R_{2i-1}\,(\delta_{ij}=0 \text{ for } i\neq j, \text{ and } \delta_{ij}=1 \text{ for } i=j),$$

$$x_i=I_{2i-1},\quad y_i=-I_p\frac{\mathrm{d}}{\mathrm{d}t}M_{p,2i-1},$$

$$I_{2i}=-I_{2i-1},\quad R_{2i}=R_{2i-1}.$$

When we express the fundamental solution of the homogeneous linear differential equation of $\mathrm{d}\boldsymbol{x}/\mathrm{d}t=\mathbf{C}\cdot\boldsymbol{x}$ $(\mathbf{C}=-\mathbf{A}^{-1}\cdot\mathbf{B})$ as $X_i(t)$ $(i=1, 2,\cdots, N_p)$, the solution of eq. (2) is

$$\boldsymbol{x}=\mathbf{X}(t)\cdot\mathbf{X}^{-1}(t_0)\cdot\boldsymbol{x}_0+\mathbf{X}(t)\int_{t_0}^{t}\mathbf{X}^{-1}(\tau)\cdot\boldsymbol{b}(\tau)\,\mathrm{d}\tau, \tag{3}$$

where

$\boldsymbol{x}=\boldsymbol{x}_0$ at $t=t_0$, and $\mathbf{X}(t)=(X_1, X_2,\cdots, X_{N_p})=\mathrm{e}^{t\mathbf{C}}\cdot\mathbf{P}=\mathbf{P}\cdot\mathrm{e}^{t\Lambda}$, $\boldsymbol{b}=\mathbf{A}^{-1}\cdot\boldsymbol{y}$.

($\mathbf{P}$: regular constant matrix, $\Lambda=\begin{pmatrix}\Lambda_1 & & & 0\\ & \Lambda_2 & & \\ & & \ddots & \\ 0 & & & \Lambda_p\end{pmatrix}$

(Jordan's normal form), $\Lambda_k=\begin{pmatrix}\lambda_k & 1 & & 0\\ & \lambda_k & \ddots & \\ & & \ddots & 1\\ 0 & & & \lambda_k\end{pmatrix}$

λ_k: eigen value of $\mathbf{C}$).

When the plasma column displaces rigidly along the z axis as δz ($0<\delta z\ll$plasma radius), $(\mathrm{d}/\mathrm{d}t)M_{p,2i-1}$ is expressed as $g_i(\mathrm{d}(\delta z)/\mathrm{d}t)$, where $g_i=(d/dz)M_{p,2i-1}$. If the time is too short compared with all of $|\lambda_i|^{-1}$, and $\boldsymbol{x}_0=\boldsymbol{o}$ at $t_0=0$ (initial induced current is zero), eq. (3) is represented as

$$\boldsymbol{x}=-\boldsymbol{h}I_p(\delta z), \tag{4}$$

where $\boldsymbol{h}=\mathbf{A}^{-1}\cdot\boldsymbol{g}$.

Then, the z component of the stabilizing force acting on the plasma column from the induced current is,

$$F_z^{(s)}=\mu_0 I_p^2(\delta z)\sum_{i=1}^{2N_p} f_i h_i, \tag{5}$$

where

$$f_i=\frac{-z_i}{\sqrt{(r_p+r_i)^2+z_i^2}}\times\left[-K(k_i)+\frac{r_p^2+r_i^2+z_i^2}{(r_p-r_i)^2+z_i^2}E(k_i)\right],$$

r_p: major radius of the plasma, r_i: major radius of the i-th coil, μ_0: permeability in the vacuum, z_i: distance along the z axis from the plasma center to the i-th coil, $k^2=(4r_pr_i/((r_p+r_i)^2+z_i^2))$. The functions K and E are the first and second kinds of the complete elliptic integrals, respectively.

When the decay index n is below zero, the destabilizing force against the z direction is

$$F_z^{(d)}=2\pi I_p B_\perp n(\delta z). \tag{6}$$

In the case of the low beta value, the vertical field $B_\perp$ is approximated as

$$-\frac{\mu_0 I_p}{4\pi r_p}\left(\ln\frac{8r_p}{a_p}+\beta_p+\frac{l_i}{2}-\frac{3}{2}\right),$$

β_p: poloidal beta value, l_i: internal inductance of the plasma, a_p: minor radius of the plasma). When the condition $F_z^{(s)}+F_z^{(d)}\leq 0$ is satisfied, the plasma is stable against the vertical displacement within the skin time. The critical value of n is

$$n_c=\frac{2r_p}{s}\sum_{i=1}^{2N_p} f_i h_i, \tag{7}$$

where

$$s=\ln\left(\frac{8r_p}{a_p}\right)+\beta_p+\frac{l_i}{2}-\frac{3}{2}.$$

Now we consider the growth rate of the positional instability. The equation of $M_p(\mathrm{d}^2/\mathrm{d}t^2)(\delta z)=F_z^{(s)}+F_z^{(d)}$ is written by

$$M_p\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\delta z)=-\mu_0 I_p^2(\delta z)\frac{s}{2r_p}(n-n_c). \tag{8}$$

(M_p: mass of the plasma). In the case of $n<n_c$, $\delta z(0)=0$,

$$\delta z(t)=d(\mathrm{e}^{\gamma_g t}-\mathrm{e}^{-\gamma_g t}), \tag{9}$$

where

$$\gamma_g=\sqrt{\frac{-s}{2r_p M_p}(n-n_c)\mu_0 I_p^2},\quad d=\text{const}.$$

The effective skin time $\tau_e=L_e/R_e$ of the passive coils can be defined from I_e, R_e and L_e.

$$I_e=\frac{1}{2}\sum_{i=1}^{2N_p}|I_i|,\quad R_e=\frac{1}{I_e^2}\sum_{i=1}^{2N_p}R_i I_i^2,$$

$$L_e=\frac{1}{I_e^2}\sum_{i=1}^{2N_p}\sum_{j=1}^{2N_p}M_{ij}I_i I_j. \tag{10}$$

This τ_e is nearly equal to $|\lambda_{\min}|^{-1}$ ($|\lambda_{\min}|$: the minimum value for all of $|\lambda_i|$), because $\mathbf{X}(t)$ has the component $\exp(\lambda_i t)$ ($\lambda_i<0$) and the $\exp(\lambda_{\min}t)$ becomes dominant when t becomes large.

2.2 *Horizontal displacement of the plasma column*

We assume that the poloidal and toroidal fluxes are conserved when the plasma column moves horizontally. The poloidal flux conservation is represented as

$$\frac{\partial}{\partial r_p}(L_p I_p)+2\pi r_p B_\perp=0, \tag{11}$$

where

$$L_p=\mu_0 r_p\left(\ln\left(\frac{8r_p}{a_p}\right)+\frac{l_i}{2}-2\right)$$

(self inductance of the plasma column). When the condition

$$\ln\left(\frac{8r_p}{a_p}\right)\gg\left|\beta_p+\frac{l_i}{2}-2\right|$$

is satisfied, eq. (11) is

$$\frac{I_p}{2}+r_p\frac{\partial I_p}{\partial r_p}=0. \tag{12}$$

From eq. (12) and the toroidal flux conservation of $a_p^2/r_p=$const. with $a_p/r_p\ll 1$,

$$\frac{\mathrm{d}}{\mathrm{d}t}(M_{pi}I_p)=I_p\frac{\mathrm{d}r_p}{\mathrm{d}t}\left(\frac{\partial M_{pi}}{\partial r_p}-\frac{M_{pi}}{r_p}\right). \tag{13}$$

When each of two coils are connected in parallel to act as a restoring force on the horizontal displacement of the plasma column (Fig. 1(b) is a example with $N_p=3$), the differential equation is

$$\mathbf{A}'\cdot\frac{\mathrm{d}\boldsymbol{x}'}{\mathrm{d}t}+\mathbf{B}'\cdot\boldsymbol{x}'=\boldsymbol{y}', \tag{14}$$

where

$$A'_{ij}=M_{2i-1,2j-1}-M_{2i-1,2j}-M_{2i,2j-1}+M_{2i,2j},$$

$$B'_{ij}=\delta_{ij}(R_{2i-1}+R_{2i}),$$

$$x'_i=I_{2i-1},$$

$$y'_i=I_p\frac{\mathrm{d}r_p}{\mathrm{d}t}\left(\frac{M_{p,2i-1}}{r_p}-\frac{M_{p,2i}}{r_p}-\frac{\partial M_{p,2i-1}}{\partial r_p}+\frac{\partial M_{p,2i}}{\partial r_p}\right),$$

$$I_{2i}=-I.$$

If the plasma column shifts horizontally as δr ($0<\delta r\ll$plasma radius), the solution of eq. (14) is derived in the same way as subsection 2.1 if the time is shorter than all of $|\lambda'_i|^{-1}$ (λ'_i: eigen value of $-\mathbf{A}'^{-1}\cdot\mathbf{B}'$).

$$\boldsymbol{x}'=-\boldsymbol{h}'I_p(\delta r), \tag{15}$$

where

$$\boldsymbol{g}'=-\frac{\boldsymbol{y}'}{I_p\frac{\mathrm{d}}{\mathrm{d}t}(\delta r_p)},\quad \boldsymbol{h}'=\mathbf{A}'^{-1}\cdot\boldsymbol{g}'.$$

Then the r component of the stabilizing force acting on the plasma column from the induced current is

$$F_r^{(s)}=\mu_0 I_p^2(\delta r)\sum_{i=1}^{2N_p}f'_i h'_i, \tag{16}$$

where

$$f_i' = \frac{-r_p}{[(r_i+r_p)^2+z_i^2]^2} \times \left[K(k_i) + \frac{r_i^2-r_p^2-z_i^2}{(r_i-r_p)^2+z_i^2} E(k_i) \right].$$

If the decay index is above 1.5, the destabilizing force is

$$F_r^{(d)} = -2\pi I_p B_\perp \left(n - \frac{3}{2} \right)(\delta r). \qquad (17)$$

The criterion of the decay index from the condition $F_r^{(s)} + F_r^{(d)} \leq 0$ is

$$n_c = \frac{2r_p}{\ln\left(\frac{8r_p}{a_p}\right)} \sum_{i=1}^{2N_p} f_i' h_i' + \frac{3}{2}. \qquad (18)$$

In the case of $n > n_c$, $\delta r(t)$ has the same form of eq. (9).

$$\delta r(t) = d'(e^{\gamma_g' t} - e^{-\gamma_g' t}), \qquad (19)$$

where

$$\gamma_g' = \sqrt{\frac{\ln\left(\frac{8r_p}{a_p}\right)}{2r_p M_p} \mu_0 I_p^2 (n-n_c)}, \quad d' = \text{const.}$$

The effective skin time also has the form $\tau_e = L_e/R_e$ and L_e, R_e are given by eq. (10).

§3. Application

3.1 *Large tokamak*

We consider here the circular plasma. But this method is easily applicable to the case of the non-circular cross section. We think the case that $a_p = 1.0$ m, $r_p = 3.0$ m and the conductivity σ of the coils is 5×10^7 $(\Omega\text{m})^{-1}$.

Figure 1 is the example of the coil position on the 'circle' (●), and on the 'rectangle' of $\kappa = 2$ (κ: height to width ratio) (○) in the case of $N_p = 3$. Each of two coils ((1, 2), (3, 4), (5, 6)) are connected in parallel, and Fig. 1(a) is drawn in the case of the vertical displacement of the plasma and Fig. 1(b) is that of the horizontal one. In the case of 'circle' in Figs. (1–5), passive coils are located around the plasma on the concentric circle a_c of 1.1 m radius, and each coil (minor radius a_m of cross section is 0.01 m) is separated from the next coil with the same angle on that circle. We use κ when the passive coils are on the rectangle (2.2 m wide, 2.2κ m high), and the distance between the neighboured coils along the side on that rectangle is same.

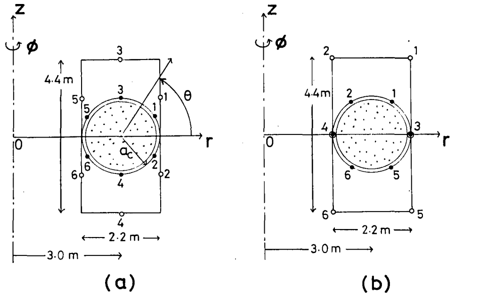

Fig. 1. Example of the coil position on the 'circle' (●), and on the 'rectangle' of $\kappa = 2$ (κ: height to width ratio) (○) in the case of N_p (number of pair coils)=3. Each of two coils ((1, 2), (3, 4), (5, 6)) are connected in parallel, and Fig. 1(a) is drawn in the case of the vertical displacement of the plasma and Fig. 1(b) is that of the horizontal one.

Figures 2 and 3 show the critical value n_c and skin time τ_e together with τ_{cal} against the vertical motion of the plasma on N_p for various configuration of the passive coils. Here, τ_{cal} is the effective value from the analytic skin time of the straight cylinder. ($\tau_{\text{cal}} = \sigma\mu_0 a_c d/2$, $d = (a_m^2/a_c)N_p$ (thickness)) When N_p becomes large ($N_p \geq 30$), n_c is saturated because the passive coils acts almost as a shell, and τ_e is nearly proportional to N_p. In the case of 'circle' with square cross section of conductors (0.01 m × 0.01 m), τ_e is nearly equal to τ_{cal} when N_p and r_p are large with $a_p = 1.0$ m fixed. The small difference is due to the finite aspect ratio $A = r_p/a_p = 3$ and infinite one. Figures 4 and 5 also show n_c and τ_e together with τ_{cal} against the horizontal motion. In these figures, we approximate

$$M_{pi} = \mu_0 \sqrt{r_p r_i} \left[\left(\frac{2}{k_i} - k_i \right) K(k_i) - \frac{2}{k_i} E(k_i) \right],$$

$$s = \ln\left(\frac{8r_p}{a_p}\right).$$

From these figures, n_c and τ_e become larger with N_p, and the value of n_c saturates, i.e. about 90% of attainable n_c (passive coils are located closely without a blank) is obtained in Fig. 2 in the case of 'circle,' when we use 16 pairs of coils which surround less than 10% of the surface ($a_c = 1.1$ m). For the vertical motion, n_c is greatly affected according to the coil position, i.e. κ, but affected small for the horizontal motion owing to the little change of the effective distance between the plasma and coils. When coils are located on the 'circle,'

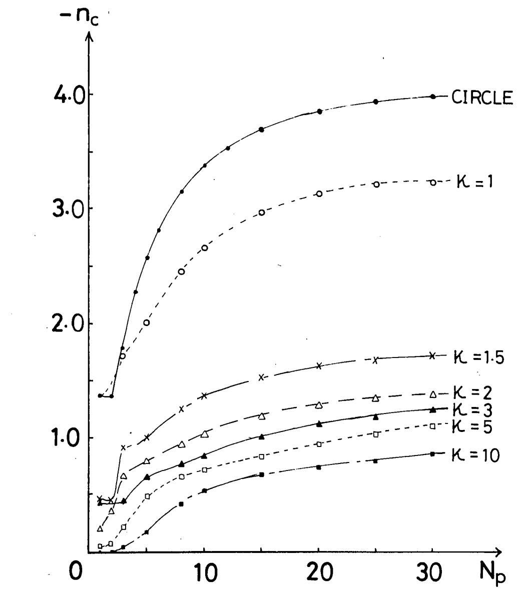

Fig. 2. n_c vs N_p for the various configurations of the passive coils against the vertical motion of the plasma.

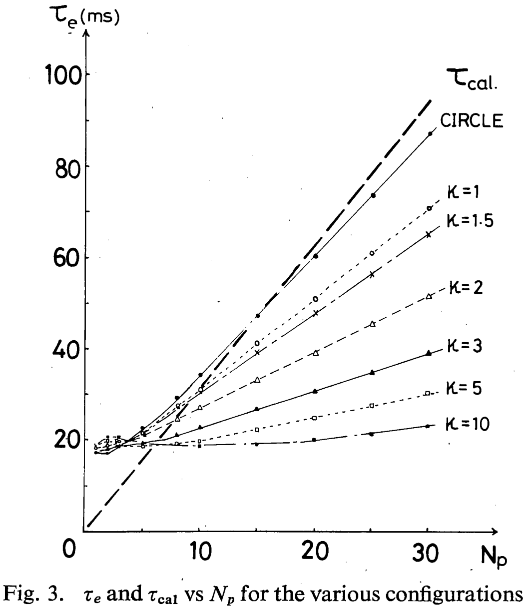

Fig. 3. τ_e and τ_{cal} vs N_p for the various configurations of the passive coils against the vertical motion of the plasma.

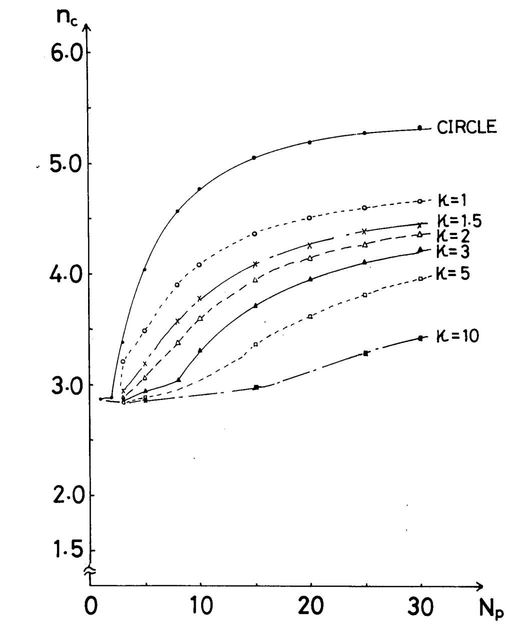

Fig. 4. n_c vs N_p for the various configurations of the passive coils against the horizontal motion of the plasma.

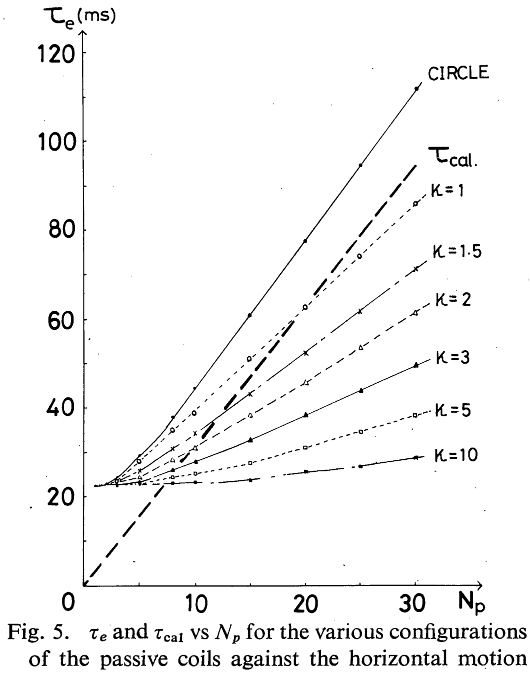

Fig. 5. τ_e and τ_{cal} vs N_p for the various configurations of the passive coils against the horizontal motion of the plasma.

the change of n_c on N_p against the vertical displacement is almost same with that against the horizontal one. In the case of the highly elongated plasma, a circle of coil locations is not outside the plasma. In that case, we must change the location of the coil.

Figure 6 shows the dependence of n_c on a_c for various values of N_p in the case of the vertical motion. As the value of $|n_c|$ becomes smaller with the increase of a_c ($|n_c| \propto a_c^{-2.5}$ and τ_e is almost constant), we must locate the coils near the plasma to get larger $|n_c|$. (When the aspect

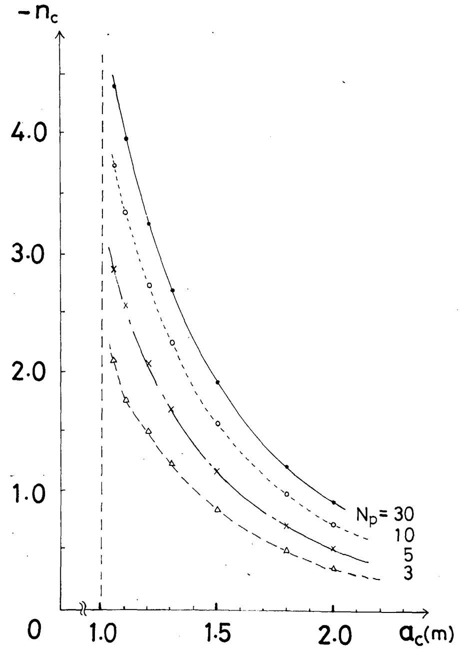

Fig. 6. n_c vs a_c for the various values of N_p in the case of the vertical motion of the plasma.

ratio becomes infinite, $|n_c| \propto a_c^{-2}$ is obtained.) Of course, $|n_c|$ reduces when the coils are outside a separatrix, which is produced by the plasma current, and the coil currents to make the vertical field.

The approximate dependence $|n_c| \propto A^{1.8}$ (the aspect ratio $A=r_p/a_p$ with $a_p=1.0$ m fixed) is obtained with $a_c=1.1$ m, $N_p=10$ for the vertical motion. The value of τ_e is from 32 to 38 ms when A changes from 2 to 100.

The relation between τ_e and a_m with $a_p=1.0$ m, $r_p=3.0$ m and $N_p=10$ for the vertical motion are investigated. Each coil is on the concentric circle a_c of 1.1 m and separated from the next coil with the same angle on this circle. The value of τ_e increases almost linearly with a_m^2 and n_c is from -2.8 to -4.3 when a_m changes from 0.001 to 0.1 m.

In the case of $a_p=1.0$ m, $r_p=3.0$ m, $a_c=1.1$ m, $a_m=0.01$ m and $N_p=173$ (coils are closely located), $n_c \sim -4.1$ and $\tau_e \sim 480$ ms is obtained. If the shell (the major radius r_p is 3.0 m, the minor radius b is 1.1 m, thickness d is 0.02 m) is divided into many coils ($N_p=173$), $n_c \sim -4.1$, $\tau_e \sim 480 \times (4/\pi) \sim 610$ ms as τ_e is nearly proportional to the cross section of the coils. The analytic skin time $\tau_s=\sigma\mu_0 bd/2$ is 680 m with $\sigma=5\times10^7$ $(\Omega\mathrm{m})^{-1}$, $b=1.1$ m, and $d=0.02$ m, in the case of the straight cylinder.

From Figs. 2–6, we must choose mainly the proper values of a_c, κ and N_p to get the needed n_c and choose a_m and N_p to get τ_e, which are decided by the plasma parameters, the discharge duration, and for example, the sampling time of the active feedback control.

Next we consider the optimum position of the passive coils to make $|n_c|$ large against the vertical motion of the plasma. By the energy principle, Rebhan and Salat[17] calculated the optimum positions of the feedback coils, too. We assume that each of two coils, symmetrically positioned with respect to the equatorial plane, are connected in parallel with a_c fixed. Figure 7 shows the dependence of n_c, τ_e and the induced current I on the angle θ (see Fig. 1(a)) when the coils are located with $\pm\theta$ ($N_p=1$), $a_c=1.1$ m, $a_p=1.0$ m, $r_p=3.0$ m, $a_m=0.01$ m and $\sigma=5\times10^{-7}$ $(\Omega\mathrm{m})^{-1}$. As is shown in this figure, $\theta\sim88°$ is the optimum value for n_c, $\theta\sim80°$ for τ_e and $\theta\sim100°$ for the induced current. In the case of $N_p=2$, the relation between n_c, τ_e and the angle α ($\theta=\pm(90°+\alpha°)$, $\pm(90°-\alpha°)$) is calculated, and $\alpha\sim15°$ is the optimum value for n_c while τ_e gradually decreases with α. Figure 8 shows the dependence of n_c and τ_e on the angle β($N_p=3$, $\theta=\pm90°$, $\pm(90°+\beta°)$, $\pm(90°-\beta°)$), and the optimum value of β

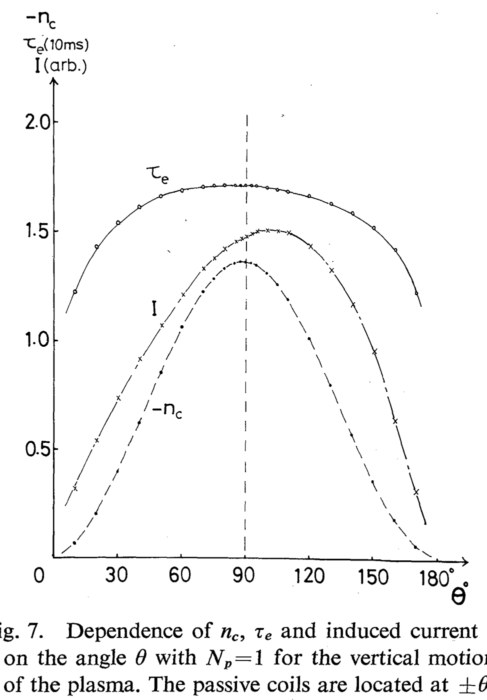

Fig. 7. Dependence of n_c, τ_e and induced current I on the angle θ with $N_p=1$ for the vertical motion of the plasma. The passive coils are located at $\pm\theta$.

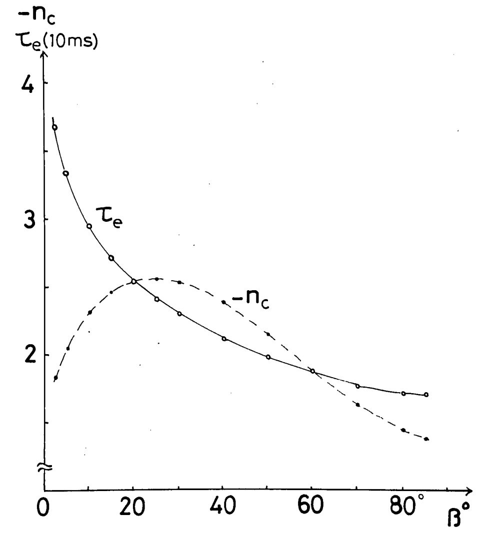

Fig. 8. Dependence of n_c and τ_e on the angle β with $N_p=3$ in the case of the vertical motion of the plasma. The passive coils are located at $\theta=\pm 90°$, $\pm(90°+\beta°)$, $\pm(90°-\beta°)$.

is 25° for n_c, while τ_e gradually decreases with β.

3.2 TOSCA

TOSCA[12,20-23] is a small tokamak to study the nature of non-circular cross sections. Robinson and Wootton[23] calculate the effects of the passive coils[18] and have a good agreement with the experimental result.

In the case of TOSCA, we put the value $r_p=30$ cm, $a_p=8$ cm, $a_m=0.75$ cm, $\sigma=5\times 10^7$ $(\Omega\mathrm{m})^{-1}$, $\beta_p=0.5$ and $l_i=0.5$ into eqs. (7), (10) and (18).

First we consider the vertical motion of the plasma. When the coils of number 2 and 15, 3 and 14, 5 and 12 from Fig. 1 in ref. 12 are connected in parallel ($N_p=3$), $n_c=-2.5$ and $\tau_e=8.3$ ms. This calculation agrees with the experimental data of $n_c\sim -2.9$ fairly well. As τ_e is greater than the discharge duration time of ~ 1 ms, the passive coils can be regarded as perfect conductors and the plasma is stable when $n>n_c$ is satisfied. In the case of the parallel connections between 2 and 15 coils, n_c is -0.62, τ_e is 5.6 ms, 3 and 14 coils make $n_c=-1.2$ and $\tau_e=6.1$ ms, 5 and 12 coils make $n_c=-1.6$ and $\tau_e=6.2$ ms. When all 16 primary windings are connected in parallel (pair coils are symmetrically located at $z=0$ plane and $N_p=8$), $n_c=-3.2$ and $\tau_e=13$ ms. In the case of $N_p=27$ (closely located), $n_c=-3.5$ and $\tau_e=40$ ms.

Secondly, we consider the horizontal motion of the plasma. When the parallel connections between 2 and 15, 12 and 15 coils and $s=\ln(8r_p/a_p)$ are taken, $n_c=2.4$ and $\tau_e=7.8$ ms. When all 16 windings are connected in parallel (pair coils are symmetrically located at $r=r_p$ plane and $N_p=8$), $n_c=3.9$ and $\tau_e=16$ ms. In the case of $N_p=27$ (closely located), $n_c=4.1$ and $\tau_e=51$ ms.

3.3 TNT-A

TNT-A[24,25] is a non-circular cross section tokamak with $r_p=40$ cm. The discharge duration time is 10–20 ms. The vacuum vessel of stainless steel is 24 cm wide and 60 cm high, and its thickness is 0.6 and 0.8 cm. The eight shaping coils are located around the chamber. In order to make vertically elongated plasmas, four outer shaping coils are used as active coils, and the top and the bottom four coils are used as passive coils. The copper shell on inside only, 10 mm thick, restricts the magnetic surface.

First we calculate n_c and τ_e for the vertical displacement of the plasma. We use the value of $a_p=10$ cm and $s=\ln(8r_p/a_p)$.

We consider the effect of the passive coils of the shaping coils. When S_1 and S_8 coils ($r=30$ cm, $z=\pm 46.9$ cm) are connected in parallel and $\sigma=5\times 10^7$ $(\Omega\mathrm{m})^{-1}$, $n_c=-0.03$ and $\tau_e=7.8$ ms. When S_2 and S_7 coils ($r=50$ cm, $z=\pm 46.9$ cm) are used as the passive coils with $\sigma=5\times 10^7$ $(\Omega\mathrm{m})^{-1}$, $n_c=-0.05$ and $\tau_e=8.7$ ms. In addition to S_1 and S_8 coils, S_2 and S_7 coils are connected in parallel ($N_p=2$), $n_c=-0.07$ and $\tau_e=9.9$ ms. This shows that these coils have a little effect on stabilizing the vertical motion of the plasma as they are relatively far from the plasma.

If we think the vacuum vessel is made of many conductors with $\sigma=1.4\times 10^6$ $(\Omega\mathrm{m})^{-1}$, $\kappa=2.5$, the values of n_c and τ_e are calculated. In the case of $a_m=0.4$ cm, $N_p=105$ (coils are closely located), we obtain $n_c=-2.2$ and $\tau_e=0.55$ ms. As τ_e is nearly proportional to a_m^2 (proportional to the area of the cross section of the coil), τ_e becomes $0.55\times 4/\pi\sim 0.7$ ms when the circular cross section of the conductors changes into a square, but n_c is almost same ($n_c\sim -2.2$). The vacuum vessel has a stabilizing effect on the vertical positional instability, but that effect vanishes gradually with time because of the small value of τ_e compared with the discharge duration time.

When the shell is divided into 60 coils ($N_p=30$) with $a_m=0.5$ cm, $\sigma=5\times10^7$ $(\Omega\text{m})^{-1}$, we have $n_c=-1.2$, $\tau_e=29$ ms. In the case of the square cross section of the coils, $n_c\sim-1.2$ and $\tau_e\sim37$ ms are obtained. This shows that the shell has not only a stabilizing effect, but also a larger value of τ_e compared with the discharge duration time in TNT-A. From the experimental data,* stable discharges are obtained when $\bar{n}_x>-1$ ($\bar{n}_x$ is the measured value when the shaping current flows) is satisfied. The above calculation can explain this experimental criterion quite well.

Secondly, the effect of the vacuum vessel against the horizontal motion of the plasma is considered. In this case, we have $n_c=6.0$ and $\tau_e=0.66$ ms. If the cross section of the conductors is square, $n_c\sim6.0$, $\tau_e\sim0.84$ ms. As the vacuum vessel is rectangle with $\kappa=2.5$, the stable region of the decay index by the passive coils against the horizontal motion of the plasma is much broader than that against the vertical one. The vacuum vessel has an influence on stabilizing the horizontal displacement of the plasma, but the skin time is shorter than the discharge duration time.

§4. Discussion and Conclusion

In the presence of the passive coils, the stabilizing force on the plasma was calculated within the time scale shorter than the effective skin time of the coils τ_e, assuming the rigid shift of the plasma column. The critical decay index n_c, the growth rate γ_g and τ_e were calculated.

As n_c is a sensitive function of the distance a_c between the plasma center and the passive coil, *e.g.* $|n_c|\propto a_c^{-2.5}$ in the case of the vertical shift of the plasma column with circular cross section (plasma major radius is 3.0 m, minor radius is 1.0 m), it is necessary to locate the coils near the plasma surface. It is also necessary to locate the coils on the optimum region, considering the angular (θ) dependence.

The skin time τ_e is proportional to the conductivity and the square of the minor radius a_m of the coil's cross section. When N_p becomes large, τ_e is nearly proportional to N_p.

So, once the required value of the decay index and the skin time of the passive coils are decided, we can choose the optimum positions of coils. That is, values of a_c, κ, N_p and θ can be determined. When the plasma shape is circular and the coils are on the side of the rectangle, $|n_c|$ decreases rapidly with κ (height to width ratio of the rectangle) against the vertical motion of the plasma column, but slowly against the horizontal one according to the effective distance between the plasma and the coils.

The values of n_c and τ_e, calculated from the above mentioned methods in two small non-circular machines of TOSCA and TNT-A, have fairly good agreements with the experimental results.

In this paper, we assume that the characteristic time is too short compared with the skin time of the passive coils. But the growth rate can be calculated when the time scale becomes large compared with the skin time, i.e. the effect of the resistance of coils can be taken into account.

In the case of $N_p=1$, the solution of eq. (3) is

$$x_1=\mathrm{e}^{-(t/\tau_e)}\int_0^t \mathrm{e}^{t/\tau_e}\frac{\mathrm{d}(\delta z)}{\mathrm{d}t}\,\mathrm{d}t.$$

Then the equation of motion is expressed as

$$M_p\frac{\mathrm{d}^2(\delta z)}{\mathrm{d}t^2}=2\mu_0 I_p^2 f_1\,\mathrm{e}^{-(t/\tau_e)}\int_0^t \mathrm{e}^{t/\tau_e}\frac{\mathrm{d}}{\mathrm{d}t}(\delta z)\,dt -\frac{\mu_0 I_p^2}{2R}sn(\delta z).$$

We differentiate this equation with respect to time,

$$\frac{\mathrm{d}^3}{\mathrm{d}t^3}(\delta z)+P_2\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\delta z)+P_1\frac{\mathrm{d}}{\mathrm{d}t}(\delta z)+P_0(\delta z)=0,$$

where

$$P_0=\Gamma^2\frac{n}{\tau_e},\quad P_1=\Gamma^2(n-n_c),$$

$$P_2=\frac{1}{\tau_e},\quad \Gamma^2=\frac{\mu_0 I_p^2 s}{2M_p r_p}.$$

This equation is reduced to the homogeneous linear differential equation

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}=\mathbf{Q}\cdot\boldsymbol{x},\quad \mathbf{Q}=\begin{pmatrix}0&1&0\\0&0&1\\-P_0&-P_1&-P_2\end{pmatrix}.$$

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In the case of $0>n>n_c$, $\tau_e\Gamma\gg 1$, the growth rate γ_g is derived as $(-n/(n-n_c))(1/\tau_e)$ from the maximum positive value of $\mathbf{Q}$. This growth rate comes from the finite conductivity of the coils. This formula explains the result of TOSCA[23] with no passive coils where $n_c\sim -6$ and $\tau_e\sim 7$ μs (the skin time of the vacuum vessel).

So far, we calculate n_c and τ_e assuming the circular cross section plasma. But in the case of the arbitrary cross section plasma, more precise value can be calculated by this method when the plasma cross section is divided into many filaments according to the plasma equilibrium.

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